

**O K L A H O M A S T A T E U N I V E R S I T Y**  
**SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING**



**ECEN 5713 Linear Systems**  
**Spring 2000**  
**Final Exam**



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**Problem 1:** (25%)

Find an “equivalent” continuous-time *Jordan-canonical-form* dynamical equation of

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 20 & 16 \\ 0 & -25 & -20 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

**Problem 2:** (25%)

For the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

determine the functions of matrix  $A$ , 1)  $A^{101}$ , and 2)  $e^{\sin At}$ .

**Problem 3:** (25%)

Determine the state transition matrix  $\Phi(t, t_0)$  for the system of equation

$$\dot{x} = e^{-At} B e^{At} x,$$

where  $A \in \mathfrak{R}^{n \times n}$  and  $B \in \mathfrak{R}^{n \times n}$ . Investigate the case when in particular  $AB = BA$ .

**Problem 4:** (25%)

Consider the system

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} x,$$

$$y = [1 \ 1 \ 1]x,$$

if possible, select initial state in such a manner so that  $y(t) = te^{-t}, t \geq 0$ .

**Problem 5:** (10% bonus)

Is it possible to transform a time-invariant system,  $\{A, B, C\}$ , into  $\{0, \bar{B}, \bar{C}\}$  by equivalence transformation?